International Trade Problem Set #7
Internal Economies to Scale

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SETUP

Suppose the world consists of two countries, A and B. Both countries can produce cars. Production costs for Q units of cars are given by $C(Q) = 100 + Q$. All firms have the same production function. Let P be the price a firm charges for its cars. Firms in both countries face an individual demand given by $Q^i(P) = S\left[\frac{1}{n} - 2(P - \bar{P})\right]$. S denotes total market size (or equivalently total demand), $\bar{P}$ denotes the average price of cars in the market, and n denotes the number of firms. Total demand in country A and B are given by $S^A = 1800$ and $S^B = 3200$.

$$\Rightarrow C^i(Q) = 100 + Q^i \ \forall i$$

$$Q^i_A(P) = 1800\left[\frac{1}{n^A} - 2(P - \bar{P}^A)\right] \ \forall i \in A$$

$$Q^i_B(P) = 3200\left[\frac{1}{n^B} - 2(P - \bar{P}^B)\right] \ \forall i \in B$$

EXERCISE 1A)

$$\Rightarrow C^i(Q) = 100 + Q^i \ \forall i$$
$$Q_A^i(P) = 1800 \left[\frac{1}{n^A} - 2(P - \bar{P}^A) \right] \ \forall i \in A$$
$$Q_B^i(P) = 3200 \left[\frac{1}{n^B} - 2(P - \bar{P}^B) \right] \ \forall i \in B$$

a) Suppose the two countries live in autarky, meaning that each country constitutes a separate market for cars. Determine the number of firms and the price of a car in each country. *Hint:* Apply the usual conditions for market equilibrium:
   1. Firms choose prices to maximize profits. Note that all firms are identical and thus they will all set the same prices.
   2. In equilibrium, firms make zero profits. This condition pins down the number of firms in the equilibrium.

EXERCISE 1A)

$$\Rightarrow C^i(Q) = 100 + Q^i \ \forall i$$
$$Q_A^i(P) = 1800 \left[\frac{1}{n^A} - 2(P - \bar{P}^A) \right] \ \forall i \in A$$
$$Q_B^i(P) = 3200 \left[\frac{1}{n^B} - 2(P - \bar{P}^B) \right] \ \forall i \in B$$

Country A:

$$\max_P \pi^i(P) = P^i Q^i(P^i) - C(Q^i(P^i))$$

$$= P^i S^A \left[\frac{1}{n^A} - 2(P^i - \bar{P}^A) \right] - 100 - S^A \left[\frac{1}{n^A} - 2(P^i - \bar{P}^A) \right]$$

$$\frac{\partial \pi(P^i)}{\partial P^i} \overset{!}{=} 0 \Rightarrow S^A \left[\frac{1}{n^A} - 2(P^i - \bar{P}^A) \right] - 2 S^A P^i + 2 S^A = 0$$

$$\Rightarrow \frac{1}{n^A} - 2(P^i - \bar{P}^A) = 2P^i - 2$$

EXERCISE 1A)

Country A:
- ▶ Realise that $\frac{1}{n^A} - 2(P^i - \bar{P}^A) = 2P^i - 2$ must hold for all i.
- ▶ $\Rightarrow P^i = \bar{P}^A$, $\forall i \in A$
- ▶ $\Rightarrow P^i = \frac{1}{2n^A} + 1$, $\forall i \in A$

▶ Make use of the fact that in equilibrium firms will make zero profit. This is due to the assumption on no entry costs. If firms would make a profit than other firms would enter the market and take those profits away.

▶ Further notice that everybody is equal: $Q^i(P^*) = \bar{Q}(P^*) = \frac{S^A}{n}$

$$\pi(P^*) = 0 : P^* \frac{S^A}{n^A} - 100 - \frac{S^A}{n^A} = 0$$
$$\frac{S^A}{2(n^A)^2} + \frac{S^A}{n^A} - 100 - \frac{S^A}{n^A} = 0$$
$$\Rightarrow S^A = 200(n^A)^2$$

EXERCISE 1A)

Country A:

$S^A = 200(n^A)^2$ (Remember $S^A = 1800$)

$\Rightarrow n^A = 3$

$\Rightarrow P^A = \frac{7}{6}$

Country B:

Realise that the problem is identical for country B and thus we obtain the same structure in the solution (Remember $S^B = 3200$)

$\Rightarrow n^B = 4$

$\Rightarrow P^B = \frac{9}{8}$

EXERCISE 1B)

b) Suppose the two countries allow for free trade in cars (meaning the two countries form one single market for cars). Determine the number of firms and the price of cars with trade. Do you think consumers are worse off or better off with trade, compared to the situation in autarky?

▶ Realise that the firms are still identical to the situation under autarky.
▶ This implies that the firms have the same maximisation problem as before:

$\Rightarrow P^{FT} = \frac{1}{2n} + 1$

$S = 200n^2$

▶ Nothing has changed on the demand side and thus $S = S^A + S^B = 5000$

$\Rightarrow n = 5$

$\Rightarrow P^B = \frac{11}{10}$

▶ The total quantity of cars remains unchanged, as still the full demand is satiated. But the price is lower under free trade and therefore the consumers will be better off.