International Trade Problem Set #4
Specific Factors Model

Armando Näf

Universität Bern

March 27, 2020

Short Refresher

Specific Factors model

A short refresher of the assumptions made in the specific factors model:

1. We have three different input factors. One input factor (labour) is used in the production of both goods, while the other two factors are **specific** for the production of one good.
2. There is perfect competition: each input factor is paid the value of its marginal product.
3. There is perfect labour mobility between the different sectors. This will have the effect that both sectors pay the same wage to labour.
4. **Exercise specific:** Production function is given by:

$$Q_C = \sqrt{L_C \cdot K}$$
$$Q_F = \sqrt{L_F \cdot T}$$

5. Produce two goods: Clothing and Food, where Capital (K) is the specific factor for clothing and Land T is the specific factor for food.

Exercise 1

Setup

$$Q_C = \sqrt{L_C \cdot K}$$
$$Q_F = \sqrt{L_F \cdot T}$$

a) Show that the marginal product of labour in each sector is decreasing in the amount of labour per specific factor in that sector?

marginal product of labour for each sector is given by:

$$\frac{\partial Q_C}{\partial L} = \frac{1}{2}\sqrt{\frac{K}{L_C}}$$
$$\frac{\partial Q_F}{\partial L} = \frac{1}{2}\sqrt{\frac{T}{L_F}}$$

Exercise 1

a) Show that the marginal product of labour in each sector is decreasing in the amount of labour per specific factor in that sector?

- marginal product of labour for each sector is given by:

$$\mathrm{MP}_L^C = \frac{\partial Q_C}{\partial L} = \frac{1}{2}\sqrt{\frac{K}{L_C}}$$

$$\mathrm{MP}_L^F = \frac{\partial Q_F}{\partial L} = \frac{1}{2}\sqrt{\frac{T}{L_F}}$$

- Taking second derivatives we show it's decreasing in L:

$$\frac{\partial \mathrm{MP}_L^C =}{\partial L} = \frac{\partial^2 Q_C}{\partial L^2} = -\frac{1}{4}\sqrt{\frac{K}{L_C}}\frac{1}{L_C} < 0$$

$$\frac{\partial \mathrm{MP}_L^F =}{\partial L}\frac{\partial^2 Q_F}{\partial L^2} = -\frac{1}{4}\sqrt{\frac{T}{L_F}}\frac{1}{L_F} < 0$$

Exercise 1

b) Show that the marginal product of each specific factor is increasing in the amount of labour per specific factor?

- marginal product of each specific factor is given by:

$$\text{MP}_K^C = \frac{\partial Q_C}{\partial K} = \frac{1}{2}\sqrt{\frac{L_C}{K}}$$

$$\text{MP}_T^F = \frac{\partial Q_F}{\partial T} = \frac{1}{2}\sqrt{\frac{L_F}{T}}$$

- Taking cross derivatives we show the marginal products are increasing in L:

$$\frac{\partial \text{MP}_K^C =}{\partial L} = \frac{\partial^2 Q_C}{\partial K \partial L} = \frac{1}{4}\sqrt{\frac{1}{K \cdot L_C}} > 0$$

$$\frac{\partial \text{MP}_T^F =}{\partial L} \frac{\partial^2 Q_F}{\partial T \partial L} = \frac{1}{4}\sqrt{\frac{1}{T \cdot L_F}} > 0$$

Exercise 1

c) Show that an increase in the relative world price of clothing ($\frac{P_C}{P_F}$) leads to a shift away from food into clothing production (i.e. show that $\frac{L_C}{L_F}$ is increasing in $\frac{P_C}{P_F}$)?

- from perfect competition we know that each factor is paid the value of its marginal product:

$$w_C = \frac{\partial Q_C}{\partial L} \cdot P_C$$
$$w_F = \frac{\partial Q_F}{\partial L} \cdot P_F$$

- From perfect labor mobility we know that the wage must be equal across sectors: $w_C = w_F$.

$$\frac{\partial Q_C}{\partial L} \cdot P_C = \frac{\partial Q_F}{\partial L} \cdot P_F$$
$$\frac{\frac{\partial Q_F}{\partial L}}{\frac{\partial Q_C}{\partial L}} = \frac{P_C}{P_F}$$

Exercise 1

c) Show that an increase in the relative world price of clothing ($\frac{P_C}{P_F}$) leads to a shift away from food into clothing production (i.e. show that $\frac{L_C}{L_F}$ is increasing in $\frac{P_C}{P_F}$)?

- Plugging in the results we have obtained for the marginal product we find:

$$\frac{\frac{\partial Q_F}{\partial L}}{\frac{\partial Q_C}{\partial L}} = \frac{P_C}{P_F}$$

$$\sqrt{\frac{T}{K}}\sqrt{\frac{L_C}{L_F}} = \frac{P_C}{P_F}$$

$$\frac{L_C}{L_F} = \left(\frac{P_C}{P_F}\right)^2 \frac{K}{T}$$

- We immediately see that $\frac{L_C}{L_F}$ is increasing in $\frac{P_C}{P_F}$. We can show this formally:

$$\frac{\partial \frac{L_C}{L_F}}{\partial \frac{P_C}{P_F}} = 2\frac{P_C}{P_F}\frac{K}{T} > 0$$

Exercise 1

d) Show that an increase in land leads to a shift of labour away from clothing into food production (i.e. show that $\frac{L_C}{L_F}$ is decreasing in T)?

- In question c) we have derived an expression for $\frac{L_C}{L_F}$:

$$\frac{L_C}{L_F} = \left(\frac{P_C}{P_F}\right)^2 \frac{K}{T}$$

- If we take the derivative of this expression with respect to T we find the following:

$$\frac{\partial \frac{L_C}{L_F}}{\partial T} = -\left(\frac{P_C}{P_F}\right)^2 \frac{K}{T^2} < 0$$

- Thus we have shown that $\frac{L_C}{L_F}$ is decreasing in T.

Exercise 2

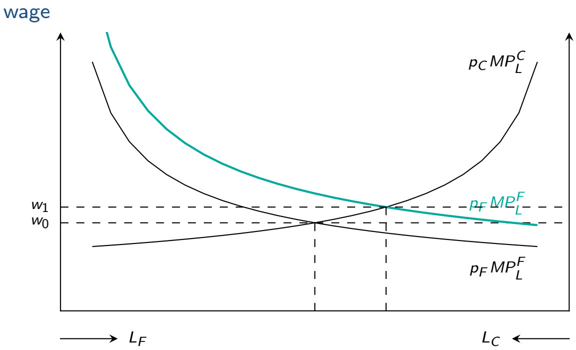
Setup

- Country is engaging in trade and produces two goods: food and clothing
- Capital is specific for clothing and land is specific for food.
- Labour is needed in the production of both goods.

a) Show graphically what happens to wages and production if the world price of food increases. What happens to *real wages* in this case? (draw a diagram as the one on the second page of the lecture notes, for simplicity I use the same production function as in a) but any function could be used.)

Exercise 2

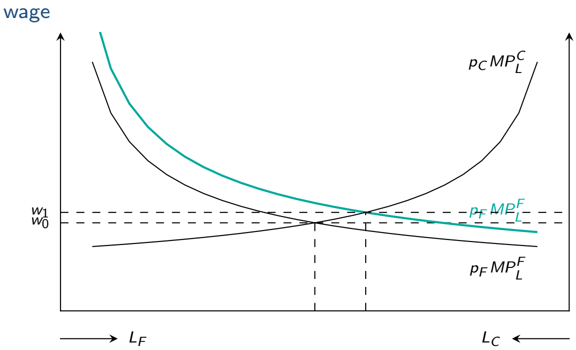
a) Show graphically what happens to wages and production if the world price of food increases. What happens to *real wages* in this case? (draw a diagram as the one on the second page of the lecture notes, for simplicity I use the same production function as in a) but any function could be used.)



- An increase of the price of food, increases the value of a marginal unit of labour in the food sector.
- Did the real wage increase?
- Nominal wage has increased but so has the price of food. Hence we don't know if the real wage has increased, remained constant or even decreased.

Exercise 2

b) Show graphically what happens to wages and production if the supply of land increases. What happens to *real wages* in this case? (Again I use the same production function as in a) but any function could be used.)



- An increase in the supply of land, increases the marginal product of labour and hence increases the value of a marginal unit of labour in the food sector.
- Did the real wage increase?
- Yes! Prices remained constant but the nominal wage as increased

Exercise 3

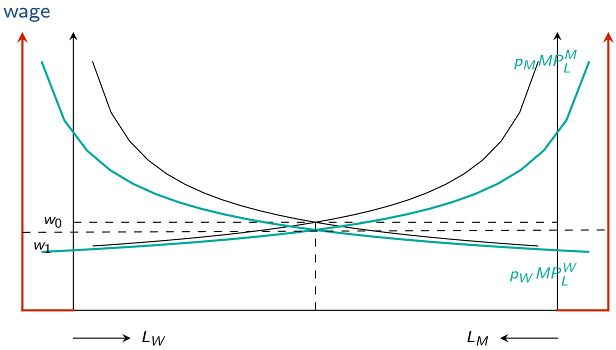
Setup

- New Zealand produces two goods: Wool and manufactured goods.
- Labour is used as an input for both goods.
- Sheep farms are the specific factor used for wool production while capital is the specific factor used in the manufacturing sector.

a) Explain in the context of the specific factors model why capitalists and farm owners in New Zealand should favour the same policies on labour immigration? Draw a diagram showing the effect of labour immigration on production and wages (for simplicity I use the same production function as in a) but any function could be used).

Exercise 3

a) Explain in the context of the specific factors model why capitalists and farm owners in New Zealand should favour the same policies on labour immigration? Draw a diagram showing the effect of labour immigration on production and wages (for simplicity I use the same production function as in a) but any function could be used).



- Immigration leads to an increase of total supply of labour.
- Distribution of labour will be unaffected, thus in both sectors marginal product of labour will decrease.
- return on capital and return on land is given by the value of their marginal product: $r_C = p_M MP_K^M$; $r_T = p_W MP_T^M$

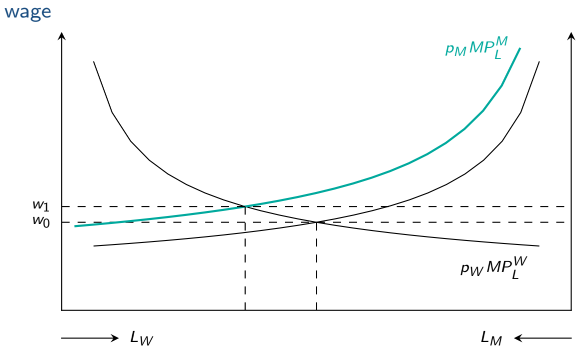
Exercise 3

a) Explain in the context of the specific factors model why capitalists and farm owners in New Zealand should favour the same policies on labour immigration? Draw a diagram showing the effect of labour immigration on production and wages (for simplicity I use the same production function as in a) but any function could be used).

- Immigration leads to an increase of total supply of labour.
- Distribution of labour will be unaffected, thus in both sectors marginal product of labour will decrease.
- return on capital and return on land is given by the value of their marginal product: $r_C = p_M MP_K^M$; $r_T = p_W MP_T^M$
- From exercise 1 a) we know that the marginal products of the specific factors are increasing in L.
- Therefore $\frac{\partial r_C}{\partial L} > 0$ and $\frac{\partial r_T}{\partial L} > 0$. This implies that both manufacturers and farm owner should be in favour of loose migration policy.

Exercise 3

b) New Zealand is a traditional exporter of wool. The government decides to introduce a tariff on imports of manufactured goods, leading to a price increase for manufactured goods. Draw a graph to show how this will affect production and wages. What do you think is the reaction of the farm owners to these tariffs? (for simplicity I use the same production function as in a) but any function could be used)



- An increase in the price for manufactured goods, will increase the value of a marginal unit of labour in the manufacturing sector.
- This will increase the nominal wage in both sectors
- Production in the manufacturing sector increases as more labour is working in that sector.
- Return on land (farm owners) will decrease as less labour is employed in that sector.

Exercise 4

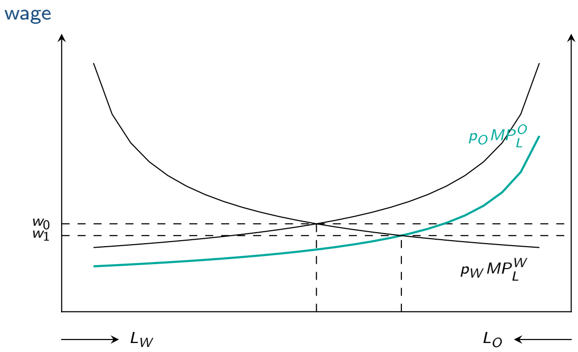
Setup

- Suppose Scotland is an independent country that produces two goods: Oil and Whiskey.
- Labour is needed to produce both goods.
- Oil fields are specific for the production of oil and whiskey factories are specific for the production of whiskey.

a) Suppose the number of oil fields decreases because some oil fields are exhausted. Draw a graph showing the effect on production and wages. What happens to the real wage? What happens to the welfare of the owners of whiskey factories and oil fields respectively?

Exercise 4

a) Suppose the number of oil fields decreases because some oil fields are exhausted. Draw a graph showing the effect on production and wages. What happens to the real wage? What happens to the welfare of the owners of whiskey factories and oil fields respectively?



- The supply of oil fields decreases and hence the marginal value of every unit of labour is going down.
- This will decrease the nominal wage in both sectors
- Production in the oil industry decreases and the whiskey sector starts producing more.
- What about the owners of the oil fields?

Exercise 4

a) Suppose the number of oil fields decreases because some oil fields are exhausted. Draw a graph showing the effect on production and wages. What happens to the real wage? What happens to the welfare of the owners of whiskey factories and oil fields respectively?

- The supply of oil fields decreases and hence the marginal value of every unit of labour is going down.
- This will decrease the nominal wage in both sectors
- Production in the oil industry decreases and the whiskey sector starts producing more.
- What about the owners of the oil fields?
- because the nominal wage has decreased, and the following equation must hold, it must be the case that the marginal product of labour has decreased.

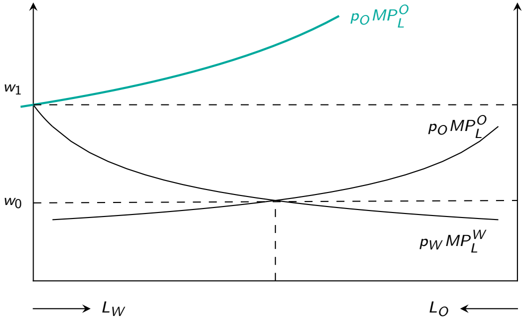
$$w_O = MP_L^O \cdot P_O$$

- since the marginal product of labour has decreased in the oil industry, it must be that there are more people employed per oil field. Thus the owners of the remaining oil fields have a higher return on their input factor.

Exercise 4

b) Suppose the world price for oil increases a lot. Show graphically that, in an extreme case, this may lead to a (almost) complete stop of whiskey production. (This phenomen is called *Dutch disease* - why is it called like that?)

wage



- Big enough increase in the oil price will increase the value of the MP of labour in the oil industry to a level where the whiskey factories will not be able to employ any workers.
- *Dutch Disease:* If one export sector can pay such high wages that another industry may be put out of business. Named because of the gas industry in the Netherlands in the 60's.